[This question paper contains 8 printed pages.]

(20) **Your Roll No.**............

Sr. No. of Question Paper : 4514

Unique Paper Code : 32221601

Name of the Paper : Electromagnetic Theory

Name of the Course : **B.Sc. Hons. – (Physics)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question No. **1** is compulsory.

3. Answer any **four** of the remaining **six**.

4. Use of non-programmable scientific calculator is allowed.

1. Attempt any **five** questions :

(a) How do you optically distinguish between quarter wave plate and half wave plate.

P.T.O.

(b) A polarimeter tube of 25 Cm long containing a sugar solution of unknown concentration rotates the plane of polarization of electromagnetic wave by 10 degrees. Specific rotation of sugar is given as 60 degrees per decimeter/gm/cc, find the concentration of the sugar solution.

(c) In case of electric field vector E be perpendicular to the plane of incidence, find the reflection and transmission coefficient for normal incidence on a air-glass interface ($n_1 = 1$, $n_2 = 1.5$).

(d) The conduction current density in a dielectric is given by J=0.02 Sin (10^9 t) Amp/m^2. Find the displacement current density if $\sigma = 10^3$ mho/m and $\varepsilon_r = 6.5$.

(e) What is the plasma frequency and minimum penetration depth for a collision free plasma having 10^{12} electrons/m^3 ?

(f) Calculate Numerical Aperture and Acceptance angle for a fiber if $n_1 = 1.458$ and $(n_1 - n_2)/n_1 = 0.01$.

(g) Show that in a good conductor the magnetic field lags the electric field by 45°. (3×5=15)

2. (a) A plane em wave propagating in a conducting medium is characterized by the parameters ε, μ and σ and show that propagation constant is complex in this case. (8)

(b) In a homogeneous region, where $\mu_r = 1$ and $\varepsilon_r = 50$

$$E = 20\,\pi \exp i(\omega t - \beta z)\ a_x \text{ Volt/m}$$

$$H = H_0 \exp i(\omega t - \beta z)\ a_y \text{ Tesla}$$

Here a_x and a_y are unit vectors in the x and y directions. Find ω and H_0 if the wavelength is 1.78m. (4)

(c) Derive the expression of skin depth for a good conductor. (3)

3. (a) State and prove Poynting theorem for a linear isotropic homogeneous medium. Explain the physical significance of each term. What is the physical significance of Poynting vector? (10)

(b) If all the energy from a 1000 W lamp is radiated uniformly, calculate the average value of the intensities of electric and magnetic fields of radiation at a distance of 2m from the lamp. (5)

4. (a) Show that Maxwell's equations can be written as two coupled second order differential equations in terms of scalar potential V and vector potential A. What is Lorentz condition and how can these equations be uncoupled using it? (8)

(b) For the propagation of electromagnetic wave through plasma derive an expression for the cut-off frequency ω_p and explain its significance. (7)

5. (a) Discuss the phenomenon of total internal reflection on the basis of electromagnetic theory. Prove that though the wave fields do exist in the second medium yet the energy flow through the surface into the second medium is zero.

(b) An electromagnetic wave whose electric field is polarized parallel to plane of incidence, is incident from free space to non-magnetic, non-conducting medium having $\varepsilon = 3\varepsilon_0$, here the wave is not reflected back from the interface. Determine the angle of transmission. (5)

6. (a) Starting from Maxwell's equations, obtain the eigen value equation for wave propagation through an optical planar waveguide for TE mode. Write its solution for the symmetric TE mode. (8)

(b) Distinguish between a step index and graded index optical fiber. Plot the variation of the refractive index with radial distance for step index and the graded index fibers. A pulse of light propagates through 1 km length of a step index fiber having a core of refractive index 1.5 and a cladding of refractive index 1.49. Calculate the pulse dispersion suffered by light on passing through the fiber. (7)

7. (a) Derive Fresnel's formulae for wave propagation in an anisotropic medium and explain the phenomenon of double refraction with the help of this. (10)

(b) A plate of 0.10 mm thickness is used as a retardation plate. For what wavelength in the visible region (400nm – 800nm) will it act as (i) quarter wave plate and (ii) half wave plate. For calcite $n_0 = 1.5443$ and $n_e = 1.5533$. (5)

Given : $\varepsilon_0 = 8.85 \times 10^{-12}$ Farad/m

$\mu_0 = 4\pi \times 10^{-7}$ Henry/m

$c = 3 \times 10^{8}$ m/s

$m_e = 9.11 \times 10^{-31}$ kg

[This question paper contains 4 printed pages.]

(21)

Your Roll No. ....

E

Sr. No. of Question Paper : 4755

Unique Paper Code : 32227612

Name of the Paper : Nano Materials and Applications

Name of the Course : **B.Sc. Hons. Physics-CBCS_DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question no. **1** is compulsory.
4. **All** questions carry equal marks.
5. Symbols have their usual meanings.

1. Attempt any **five** questions : (3×5=15)

(a) Calculate the capacitance of the isolated spherical metals nanostructures of size 10nm, 100nm and 1000nm. What are the corresponding charging energies? ($\varepsilon_o = 8.854 \times 10^{-12}$ F/m).

P.T.O.

(b) What do you understand by density of states (DOS)? Draw the schematic comparison of the DOS of 3D and 2D nanomaterials.

(c) How many cubes with a side of 1 nm can be carved out from a cube with a side of 1 cm? Compare the surface area to volume ratio for both the cases.

(d) In a sodium chloride crystal, there is a family of planes 0.252 nm apart. Can it be called a nanomaterial? If the first-order maximum is observed at an incidence angle of 18.1 degree, what wavelength X-ray was used for its analysis?

(e) What are the disadvantages of SEM and TEM over optical microscope?

(f) Explain deep and surface level defects with a suitable diagram.

(g) What will happen when the length of a device is less than the mean free path of electron with reference to electron transport properties?

(h) The absorption spectra of SnO_2 nanoparticles in a solution is taken at a time interval at 0 sec, 20 sec and 60 sec. The absorption edge is observed at 350nm, 370nm, and 380 nm respectively. Calculate and plot of the bandgap as a function of time.

2. (a) Derive the expression of bound state energies of a particle in a cubical 3-D potential box. The potential inside the box is zero and outside it is infinity. Draw the energy states of the lowest 5

energy states mentioning the different values of n_x, n_y and n_z. Discuss the degeneracy in each energy state. (10)

(b) A bulk semiconductor has direct bandgap of 1.6 eV and the effective mass of holes in valence band and electrons in conduction band are same as that of free electron ($m_e = 9.1\times10^{-31}$ kg). If the same material of thickness 5 nm is sandwiched between material of infinite barrier height, then due to confinement the energy levels get shifted. Find the new bandgap of thin film ($h = 6.626 \times 10^{-34}$Js). (5)

3. (a) Calculate the density of states for bulk material in the energy range 0 to 2eV. Take mass of electron $= 9.1 \times 10^{-31}$kg, $h = 6.626 \times 10^{-34}$Js. Compare the result with that of 2D nanostructure. (8)

(b) Write the expression for density of states of a 0D material with explanation of each term. Compare the plots of DOS for 0D, 3D and 2D material and discuss the findings. (7)

4. Discuss the basic principle and working of any two of the following fabrication techniques used for the synthesis of nanomaterials? Discuss the various deposition parameters for each.

(i) Sol-gel

(ii) Sputtering

(iii) Hydrothermal method

(iv) Molecular Beam Epitaxy (7.5×2=15)

5. (a) Discuss with diagram the working principle of the scanning tunneling microscope (STM). How STM is superior to SEM and TEM? (9)

(b) What are the various modes of operation of the Atomic force microscope (AFM). (6)

6. (a) Discuss Coulomb Blockade effect with suitable device structure. (9)

(b) Explain the process of thermionic emission of electrons from a material. (6)

7. (a) Define direct and indirect bandgap semiconductors using E-k diagram and give at least one example and application of each. (5)

(b) What are excitons in the context of semiconductors. What do you understand by Frenkel and Mott- Wannier excitons? (10)

8. (a) Explain the following length scales of electrons in nanomaterials :

(i) Mean free path and phase breaking length.

(ii) elastic and inelastic scattering lengths. (6)

(b) What is the difference between ballistic and diffusive transport? Provide suitable diagram. (4)

(c) Obtain an expression for the variation in electrostatic energy of a system for single electron charging. Name the effect associated with it. (5)

[This question paper contains 8 printed pages.]

(22) Your Roll No. 2023

Sr. No. of Question Paper : 4762

Unique Paper Code : 32227626

Name of the Paper : Classical Dynamics (DSE – Paper)

Name of the Course : **B.Sc. (Hons.) Physics (CBCS – LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **four** questions in all including Question No. **1** which is compulsory.

1. Attempt any **four** of the following :

   (a) What are generalized coordinates? Derive expressions for the generalized displacement and generalized velocity.

(b) A particle of mass m is attached to a point by inverse square law. Find the Hamiltonian and canonical momentum.

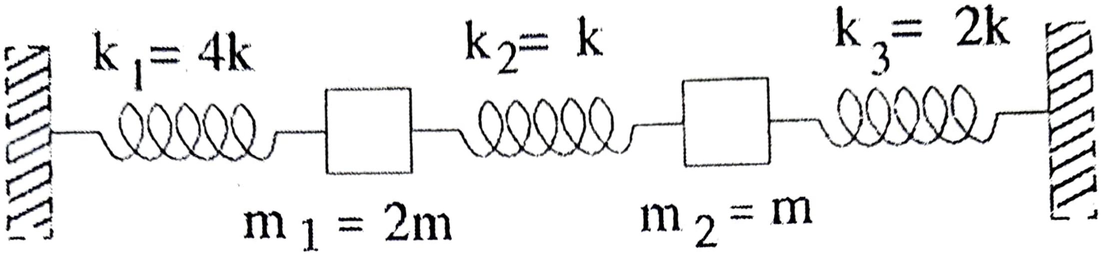
(c) In an inertial frame S, photon emitted from the light source at origin gets reflected by the mirror normal to x–axis. If the mirror is placed at $x = 7$ units, draw the world lines of mirror, emitted photon and that of reflected photon.

(d) At what speed does a clock moves if it runs at a rate which is one-half of the rate of a clock at rest?

(e) Differentiate between laminar flow and streamline flow. Give examples.

(f) Consider two masses $m_1 = 2m$ and $m_2 = m$ connected by three springs with spring constants as shown in the figure below. Find the kinetic energy (T) and potential energy (V) matrices for the system.

(4×6=24)



(4×6=24)

2. (a) State Hamilton's principle. Derive Lagrange's equation of motion from Hamilton's principle for a conservative system. (2,7)

(b) A particle of mass m moves in a central force field of potential V. Find its Hamiltonian and Hamilton's equations of motion. (4,4)

3. (a) Consider a spring pendulum: a simple pendulum in which massless string is replaced by a massless spring (unstretched length r_0, spring constant k). The bob of mass m oscillates in a vertical plane about its equilibrium position. If the instantaneous

position of this bob is (r, θ). obtain the Lagrangian of this system and hence find the equation(s) of motion.

(Assume $r > r_0$ and $\theta = 0$ is the equilibrium position) (5,4)

(b) An electron is accelerated from rest by a potential difference of 350 V. Then it enters a magnetic field $\vec{B}$ of magnitude 200 milli-Tesla with its velocity perpendicular to $\vec{B}$. Find the

(i) speed of an electron.

(ii) radius of the path in magnetic field.

(4,4)

4. (a) If $\tilde{P} = (E/c, \vec{p})$ is the 4-momentum, show that $E^2 = c^2p^2 + m_0^2 c^4$. (5)

(b) In an inertial frame S, a particle describes a path with parametric equations :

$x(t) = at + b\ \sin(\omega t);\ y(t) = b\cos(\omega t);\ z(t) = 0.$

Calculate the velocity, acceleration, 4-velocity and 4-acceleration of this particle. (2,2,4,4)

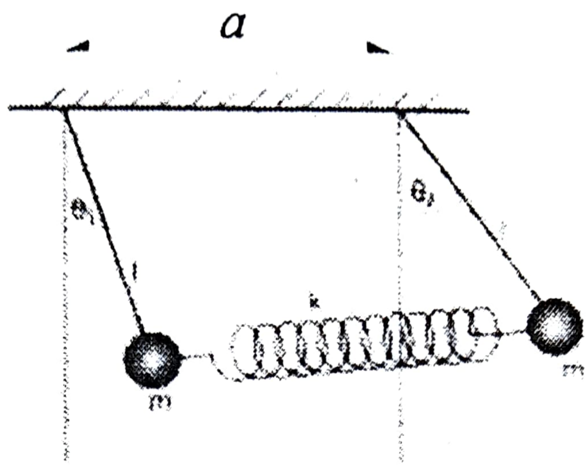
5. (a) Starting from the 4-displacement vector, $\tilde{X} = (ct, x, y, t) = (ct, \vec{r})$, prove that $\tilde{U} \bullet \tilde{A} = 0$ where $\tilde{U}$ is 4-velocity and $\tilde{A}$ is 4-acceleration. (9)

(b) In an inertial frame S, proton has 4-momentum, $\tilde{P} = (E/c, \vec{p})$ and an observer has 4-velocity, $\tilde{U} = \gamma_u (c, \vec{u})$. Show that the proton's energy, as measured by this observer is $\tilde{U} \bullet \tilde{P} = 0$, where

$$\gamma_u = \frac{1}{\sqrt{1-(u/c)^2}} \cdot \quad (4)$$

(c) An unstable atom of rest mass M_o at rest decays into two daughter atoms a (rest mass m_{oa} and speed (u_a) and b (rest mass m_{ob} and speed (u_b). Using 4-vector approach or otherwise, find the energy of daughter atom a. (4)

6. Coupled pendulums are executing simple harmonic oscillations as shown below. The generalized coordinates are (θ_1, θ_2) and the relaxed length of spring is a. Find the kinetic energy T and the potential energy V matrices, and hence find the angular frequencies of small oscillations. (5,7,5)



7. (a) A charged particle (mass m, charge q) is moving in x – y plane. At time t = 0, this particle is at the origin having velocity v_0 at an angle 45° with the x–axis and uniform crossed electric and magnetic fields : $\vec{E} = E_0\hat{y}$ and $\vec{B} = B_0\hat{z}$ are switched on. Find its velocity at time t > 0. (8)

(b) Draw the analogy between fluid flow in a pipe and current flow in a circuit. Two capillaries of radius r and R and same length are connected to each other in series, r << R. A liquid of viscosity coefficient η is flowing through them. Find an expression for the volume flow rate for the arrangement. (3,6)

[This question paper contains 8 printed pages.]

(23) **Your Roll No.** 2023

Sr. No. of Question Paper : 4764

Unique Paper Code : 32227630

Name of the Paper : Adv. Quantum Mechanics

Name of the Course : **B.Sc. Hons CBCS DES**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **Four** question in total.

3. **All** questions carry equal marks.

1. (a) If the eigenstates $\{|\lambda_i\rangle\}$ of an observable, $\hat{M}$ are all orthonormal, i.e. $\langle\lambda_i|\lambda_j\rangle = \delta_{ij}$ then there exists

P.T.O.

a unitary transformation $\hat{U}$ such that in the new basis the observable is diagonal, i.e. $\hat{U}^{\dagger}\hat{M}\hat{U}$ is diagonal.

(b) Show that if two observable $\{\hat{A}, \hat{B}\}$ commute, there exists a similarity transformation $\hat{S}$ such that $\hat{S}^{-1}\hat{A}\hat{S}$ and $\hat{S}^{-1}\hat{B}\hat{S}$ are diagonal.

(c) For a particle with the spin $s = 1/2$, find the eigenvalues and eigenfunctions of the operators $\hat{S}_x, \hat{S}_y, \hat{S}_z$. (6+6+6.75)

2. (a) What do you mean by measurement in Quantum mechanics? How it is different from measurement in classical mechanics? What do you mean by expectation value of an observable? What are

expectation values and eigenvalues for a spin half system?

(b) Find the generalized uncertainty relation for two observable A and B. Mention and prove the three useful results from linear algebra required to find the uncertainty relation. (8+10.75)

3. (a) Show that any operator $\hat{\Omega}$ can be written as $\sum_{i,j=1}^{n} \Omega_{ij} |i\rangle\langle j|$, where Ωij are the matrix elements of $\hat{\Omega}$, and $\{|i\rangle\}$ and $\{|j\rangle\}$ are the same orthonormal basis with basis vectors labelled by the indices i and j respectively.

(b) Let $|x\rangle$ and $|p\rangle$ be the eigenstates of position and momentum respectively, which satisfy $\hat{x}|x\rangle =$

$x|x\rangle$ and $\hat{p}|p\rangle = p|p\rangle$, and $\langle p|x\rangle = \exp(-i\,p\,x/\hbar)/(2\pi\hbar)$. Consider $\langle p|\hat{x}|\psi\rangle$ for an arbitrary state $|\psi\rangle$, find the explicit form of the operator $\hat{x}$ in the momentum space. (8+10.75)

4. (a) Show that the infinitesimal time evolution operator $\hat{u}(t_0 + dt, t_0) = 1 - i\hat{\Omega}\,dt$ satisfies unitary group properties. Relate the Hamiltonian of the system with time evolution. Thus find the Schrodinger equation for time evolution operator. How it is related to Schrodinger equation of state in position basis? Write the finite time evolution operator when Hamiltonian operator $\hat{H}$ at different times do not commute.

(b) Using the one-dimensional simple harmonic oscillator as an example, illustrate the difference between the Heisenberg picture and the Schrodinger picture. Discuss in particular how (i) the dynamic variables x and p and (ii) the most general state vector evolve with time in each of the two pictures. (10+8.75)

5. (a) Determine explicitly the form of the operator $\hat{S}_n$, corresponding to the spin projection in the direction determined by a unit vector $\hat{n}$ (not an operator). For a state with a definite value of the spin z-projection, determine the value of $\langle \hat{S}_n \rangle$ and find the probability of finding spin +1/2 along $\hat{n}$.

(b) Consider a spin half system under a finite rotation along the xy-plane. The generators of rotational transformation for this system are

$$\hat{S}_x = \frac{\hbar}{2}\left(|+\rangle\langle -| + |-\rangle\langle +|\right)$$

$$\hat{S}_y = \frac{i\hbar}{2}\left(-|+\rangle\langle -| + |-\rangle\langle +|\right)$$

$$\hat{S}_z = \frac{\hbar}{2}\left(|+\rangle\langle +| - |-\rangle\langle -|\right)$$

The spin state transforms as $|\alpha\rangle_R = \mathcal{D}_z(\phi)\,|\alpha\rangle$ where $\mathcal{D}_z(\phi) = \exp\left(-\frac{i\hat{S}_z\phi}{\hbar}\right)$. Show that while state $|\alpha\rangle$ transform as spinor the expectation values $\langle \hat{S}_x \rangle, \langle \hat{S}_y \rangle$, and $\langle \hat{S}_z \rangle$ transform as components of a vector. (8+10.75)

6. (a) Use Variational principle to estimate the ground state of a harmonic oscillator using the trial function

$$\phi(x) = \frac{1}{1+\beta x^2}$$

Given the hamiltonian of harmonic oscillator is

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{k}{2}x^2$$

Following integrals are given :

$$\int_{-\infty}^{\infty} \frac{dx}{(1+\beta x^2)^3} = \frac{3\pi}{8\beta^{1/2}},$$

$$\int_{-\infty}^{\infty} \frac{x^2\, dx}{(1+\beta x^2)^4} = \frac{\pi}{16\beta^{1/2}},$$

$$\int_{-\infty}^{\infty} \frac{x^2\, dx}{(1+\beta x^2)^2} = \frac{\pi}{2\beta^{3/2}}$$

(b) For a particle of mass m moving in a one dimensional box with walls at $x = 0$ and $a = L$. Use a trial function $\psi(x) = x(L - x)$ to evaluate the ground state energy of the system.

(10+8.75)

[This question paper contains 4 printed pages.]

(24)

Your Roll No. 2023

Sr. No. of Question Paper : **4794** **E**

Unique Paper Code : 32221602

Name of the Paper : Statistical Mechanics

Name of the Course : **B.Sc. (Hons) Physics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **five** questions in all.

3. **Question No. 1** is compulsory.

4. **All** questions carry equal marks.

5. **Non- programmable Scientific calculators** are allowed.

1. Attempt any ***five*** of the following:

(a) Draw a phase space for a one-dimensional classical linear harmonic oscillator of mass $m$having total energy $E = p^2/2m + m\omega^2x^2/2$. Calculate the total number of microstates available to it, if the energy of a harmonic oscillator lies between 0 and E.

(b) Discusswhether law of equipartition of energy can be applied to the following systems or not:

(i) classical harmonic oscillators

(ii) gas consisting of free particles moving non- relativistically

(iii) nucleons in a nucleus.

(c) Assume a system of N bosons, each of mass m, is confined to a one-dimensional box of length L at temperature T = 0 K. The energy levels of the system are given by $E_n = n^2h^2/(8mL^2)$, where n= 1, 2,... and $h$is Planck's constant. Find the total energy of the system in terms of N, L, and h.

(d) Plot the variation of specific heat with temperature for ideal Bose-Einstein gas and explain its behavior in the strongly degenerate and classical regions.

(e) For Silver atom, with one electron per atom at room temperature, the number density n = $5.86\times10^{28} m^{-3}$. Find the nature of the degeneracy of the system under consideration.

(f) A cubical cavity of side 1m is filled with black body radiation. Calculate the number of independent standing waves with wavelengths in the range 8.0 mm and 9.0 mm.

(g) Calculate the normal radiation pressure generated by an incandescent bulb of 200W at a distance of 1m.

(5×3=15)

2. (a) Using the partition function of classical ideal monoatomic gas consist of N indistinguishable particles at fixed temperature T in volume V: $Z\,(N,V,T) = (V^N/N!)\,[2\pi mk_BT/h^2]^{3N/2}$:

Derive the Sackur-Tetrode relation assuming N >>1. Show that the entropy given by the Sackur- Tetrode equation is an extensive parameter.

(b) A partition divides a box into two chambers 1 and 2, each of volume V. Assume that chamber 1 and 2 chamber contain the same ideal gas consisting of $2N$particlesand N particles respectively attemperature T. Using the Sackur-Tetrode relation, calculate the entropy of mixing after the partition is removed and the contents are allowed to mix to reach at equilibrium (Assume that the temperature remain constant throughout the process). (8, 7)

3. (a) Consider an isolated paramagnetic salt consisting of dipoles of magnetic moment μ,located in an external magnetic field B. Out of these N dipoles, n dipoles are parallel to B and rest are anti-parallel. Show that the total energy E, the entropy S and the absolute temperature T of the system are given respectively as

(i) $E = (N-2n)\,\mu B$

(ii) $S = -Nk_B\,[x\,ln\,(x) + (1-x)\,ln\,(1-x)]$, where $x = n/N$

(iii) $1/T = [k_B/2\,\mu B]\,ln\,(n/(N-n))$

(b) A system consists of 12 identical butdistinguishable particles which can occupy non-degenerate energy levels. Initially, the system is in the macrostate which is defined by (6,3,2,1) particles in the energy levels $(0, \varepsilon, 3\varepsilon, 5\varepsilon)$.

(i) Calculate the number of microstates and entropy of the system in its initial state.

(ii) If a small amount of energy is added to the above-mentioned system such that only one particle is raised from the ground level (zero energy) to first excited level (ε), calculate the number of microstates available in this final microstate.Hence, find the change of entropy when system undergoes from initial to final state. (8, 7)

4. (a) Consider a completely degenerate non-relativistic gas of electrons in3 –dimensions. Obtain the expressions for average energy per particle, Fermi velocity and Fermi pressure.

(b) Consider the model of a white dwarf star: a sphere consisting of helium gas of mass $M = 10^{30}\ kg$ at a density of $\rho = 10^{10}\ kg\ m^{-3}$ and temperature T of the order of $10^6 K$. Using these data, find the nature of electron gas inside a white dwarf star. (Given: mass of proton $\approx 10^{-27}\ kg$, mass of electron $\approx 10^{-30}\ kg$) (8, 7)

5. (a) Prove that for photon gas, internal energy (U) and (S) entropy at given temperature T are related by the following relation: $TS = 4U/3$.

(b) How does Bose–Einstein condensation explain the superfluid properties of liquid 4*He*?

(10,5)

6. (a) A blackbody cavity at temperature T is filled with N_0, N_1, N_2, ... oscillators having energies 0, $h\nu$, $2h\nu$, ,...respectively. Calculate the total number of oscillators and determine average energy of the oscillators. If Planck's constant tends to zero, what would be the effect on the average energy of the oscillators?

(b) Calculate the average energy of a Planck oscillator, vibrating with frequency 3×10^{14} Hz at 2000 K. Compare it with a classical oscillator. (10,5)

Constants:

$k_B = 1.38 \times 10^{-23} JK^{-1}$

$m_e = 9.11 \times 10^{-31} kg$

$h = 6.626 \times 10^{-34}\ Js$

$c = 3 \times 10^8\ ms^{-1}$

$\dot{\sigma} = 5.67 \times 10^{-8}\ J\ m^{-2} s^{-1} K^{-4}$

[This question paper contains 4 printed pages.]

(25) Your Roll No. 2023

Sr. No. of Question Paper : 4879

Unique Paper Code : 32227612

Name of the Paper : Nano Materials and Applications

Name of the Course : **B.Sc. Hons. Physics – CBCS_DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question no. **1** is compulsory.
4. **All** questions carry equal marks.
5. Symbols have their usual meanings.

1. Attempt any **five** questions : (3×5=15)

   (a) Name the nanostructures where an electron feels (i) 1 dimensional, (ii) 2 dimensional and (iii) 3-dimensional confinement. Give one example of each.

(b) What do you understand by size effect and quantum size effect? Give one example of each.

(c) What is graphene and how is it different from C_{60} and carbon nanotubes?

(d) The XRD data of a material exhibits a peak at a 2θ angle of 30 degrees and FWHM of 0.3 degrees. Comment whether the synthesized material will be called a nanomaterial or not. (X-ray wavelength used is 0.154 nm).

(e) The bandgap of $Al_XGa_{(1-X)}N$ can vary from 3.4(x=0) to 6.2(x=1) eV. What is the range of wavelengths that can be emitted from such semiconductor?

(f) The absorption edge of ZnO thin films of thicknesses 10nm, 20nm, and 50nm are observed at 350nm, 360nm, and 375nm respectively. Plot the bandgap as a function of film thickness.

(g) How nanomaterials can be used for cancer therapy?

(h) Write the expression of the bandgap of a spherical nanoparticle. Explain the effect of excitonic contribution.

2. (a) Define density of states and its units. Drive the expression for the density of states for ID structure. Compare the results with the density of states of 3D structure using suitable energy curves. (10)

(b) An electron of mass 9.1×10^{-31} kg is trapped in a cubical box each side 1A°. Calculate the eigen energy value corresponding to 3rd energy level. Is there any degeneracy in these states, if yes, determine it? ($h = 6.626 \times 10^{-34}$ Js). (5)

3. (a) Discuss the phenomenon of quantum mechanical tunneling and hopping conductivity in solids with proper band diagrams? Give one application of each. (10)

(b) An electron with kinetic energy $E = 16$ eV is incident on a potential step of height $V = 7$ eV. Calculate the reflection coefficient. (5)

4. (a) Explain with suitable diagram the process of UV photolithography technique for patterning any structure. (8)

(b) Explain the nucleation and growth process in the synthesis of colloidal nanomaterials. (7)

5. What is the basic working principle of chemical vapor deposition (CVD) technique? Discuss any one of the CVD technique in detail. What are the various parameters that needs to be controlled during deposition? (15)

6. (a) Discuss the working of scanning electron microscope (SEM) with a neat diagram. (8)

(b) What will happen if the energy of the electron is very high, say relativistic? (2)

(c) What are the advantages and disadvantages of SEM over optical microscopes. (5)

7. (a) How an exciton is classified in the context of binding energy? Derive the expression for the binding energy of an exciton. (12)

(b) The interfacial energy for barium sulphate nanocrystals in saturated aqueous solution is 120 mJ/m^2. If the critical radius is 1 nm, calculate the value of the Gibbs free energy barrier. (3)

8. (a) How is ballistic transport different from diffusive transport? Explain with the help of diagram and give one example of each. (7)

(b) Explain the working of a single electron transfer device. (8)

[This question paper contains 8 printed pages.]

(26) Your Roll No. 2023

Sr. No. of Question Paper : 4885 E

Unique Paper Code : 32227626

Name of the Paper : Classical Dynamics (DSE – Paper)

Name of the Course : **B.Sc. (Hons.) Physics (CBCS – LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **four** questions in all including Question No. **1** which is compulsory.

1. Attempt any **four** of the following :

(a) Define cyclic coordinate and show that generalized momentum conjugate to a cyclic coordinate is conserved.

(b) Find the Lagrangian corresponding to the Hamiltonian

$$H = \frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy,$$ where a, b, k are constant.

(c) Explain the phenomenon of length contraction using space-time diagram.

(d) A particle of rest mass m_0 describes a circular path with parametric equations :

$$x = a \cos t,\ y = a \sin t,\ z = 0$$

in an inertial frame S. Find the 4-velocity components of this particle and hence norm of 4-velocity in S.

(e) A particle of mass m is moving in a potential

$$V(r) = \frac{1}{4}bx^4 - \frac{1}{2}ax^2,$$ where a, b are positive constants. Obtain the equilibrium position and the angular frequency of small oscillations.

(f) Derive Poiseuille's equation for the flow of liquid through a pipe. (4×6=24)

2. (a) A charged particle (mass m, charge q) is moving in a potential V(x,y) and in a magnetic field $\vec{B} = B_o \hat{z}$. Lagrangian of the system is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V(x,y) + \frac{qB_o}{2c}(x\dot{y} - y\dot{x})$$

(i) Obtain the Lagrangian equations of motion.

(ii) Find the momenta (p_x, p_y, p_z) conjugate to (x, y, z). (6,6)

(b) A particle of mass m is falling freely in a vertical direction near the surface of the earth. Write the Lagrangian of the system and hence obtain the equation of motion. (5)

3. (a) A block A of mass m is constrained to slide without friction on a straight track on a table in earth gravity. A bob B of mass m is attached to

the block A by a massless inextensible string. Making small angle approximations, find the normal modes with their frequencies and normalized coordinates. (10)

(b) For the given Langrangian $L = \dot{x}^2/2 - \omega^2 x^2/2 - \alpha x^3$, find the corresponding Hamiltonian. Also find the equations of motion. (7)

4. Four-displacement is given by $X^\mu = (ct, xy, z) = (ct, \vec{r})$

and $\vec{u} = \frac{d\vec{r}}{dt}$, $\vec{a} = \frac{d\vec{u}}{dt}$. Show that

(a) $\frac{dt}{d\tau} = \gamma_u = \frac{1}{\sqrt{1-(u/c)^2}}$ and $\frac{d\gamma_u}{dt} = \frac{\gamma_u^3}{c^2}\vec{u}\bullet\vec{a}$, where τ is the proper time.

(b) 4-velocity is given by $U^\mu = \gamma_u(c, \vec{u})$.

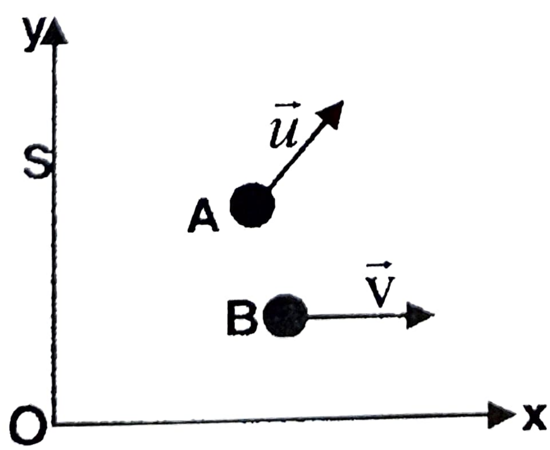
(c) 4-acceleration is given by

$$A^{\mu} = \gamma_u^4 \left(\frac{\vec{u} \cdot \vec{a}}{c}, \left(1 - \frac{u^2}{c^2}\right)\vec{a} + \frac{\vec{u} \cdot \vec{a}}{c}\vec{u} \right)$$ (7,3,7)

5. (a) In Lab frame S, particles A and B are moving with velocities $\vec{u}$ and $\vec{v}$ respectively. If $\vec{u}'$ is the velocity of A with respect to B, then show that

$$\gamma_{u'} = \gamma_u \gamma_v \left(1 - \frac{\vec{u} \cdot \vec{v}}{c^2}\right) \quad \text{where} \quad \gamma_w = \frac{1}{\sqrt{1-(w/c)^2}},$$

hence find u′ for the special cases :

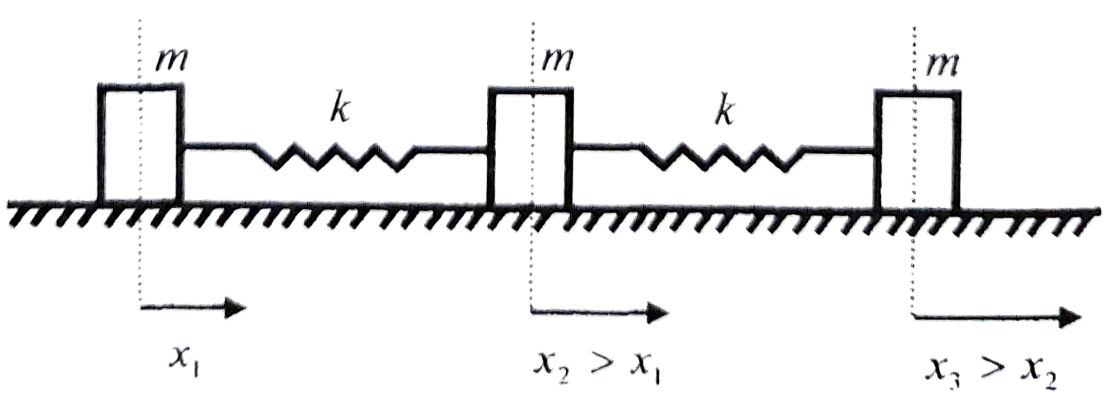


(i) $\vec{u} = u\hat{x}$ and $\hat{v} = v\hat{x}$

(ii) $\vec{u} = u\hat{y}$ and $\hat{v} = v\hat{x}$ (5,3,3)

(b) An unstable atom at rest (rest mass M_0) decays into two daughter atoms, each having rest mass m_0 and speed u, using 4-vector approach or otherwise, prove that $u = \frac{c}{M_o}\sqrt{M_o^2 - 4m_o^2}$. (6)

6. Three blocks and two springs are configured as shown below. These blocks can execute longitudinal simple harmonic oscillations only. When the blocks are at rest, all springs are unstretched.



(a) Choosing the displacement of each block from its equilibrium position as generalized coordinates (x_1, x_2, x_3), obtain T (kinetic energy) and V (potential energy) matrices and hence find the angular frequencies of small oscillations.

(3,3,3)

(b) Find the relations between (x_1, x_2, x_3) and normal coordinates (q_1, q_2, q_3) and hence obtain the expression of T in terms of (q_1, q_2, q_3). (6,2)

7. (a) Two photons (each with energy E) collide at an angle θ and create a particle of rest mass m_0. Using 4-vectors or otherwise, show that

$$m_0 = \frac{E}{c^2}\sqrt{2(1-\cos\theta)}\,.$$

(b) Derive continuity equation in 3-dimensions for incompressible fluids.

(c) Verify if the continuity equation for steady incompressible flow is satisfied or not for the following velocity components :

$$v_x = 3x^2 - xy + 2z^2,$$

$$v_y = 2x^2 - 6xy + y^2,$$

$$v_z = -2xy - yz + 2y^2$$ (6,6,5)

[This question paper contains 8 printed pages.]

(27) **Your Roll No.** ....

Sr. No. of Question Paper : 4887

Unique Paper Code : 32227630

Name of the Paper : Adv. Quantum Mechanics

Name of the Course : **B.Sc. Hons CBCS DES**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **Four** question in total.

3. **All** questions carry equal marks.

1. (a) If two Hermitian operators $\hat{A}$ and $\hat{B}$ have the same eigenvalues with the same multiplicities,

show that $\hat{A} = U^{\dagger}\hat{B}U$ for some unitary operator U.

(b) Let $\hat{A}$ and $\hat{B}$ be two observable. Suppose the simultaneous eigenket of $\hat{A}$ and $\hat{B}$, $\{|a', b'\rangle\}$ form a complete orthonormal set of base kets. Can we always conclude that $[\hat{A}, \hat{B}] = 0$? If your answer is yes, prove the assertion. If your answer is no, give a counter example.

(c) Two Hermitian operators anticommute:

$\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A} = 0$. Is it possible to have a simultaneous (that is, common) eigenket of $\hat{A}$ and $\hat{B}$? Prove or illustrate your assertion.

(6+6+6.75)

2. (a) Discuss the Gram-Schmidt process for orthonormalizing a set of linearly independent vectors in an inner product space. Suppose that we have the set

$$|\alpha_1\rangle = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad |\alpha_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad |\alpha_3\rangle = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Check if these vectors are orthonormal. If not, construct the orthonormal basis set using Gram-Schmidt scheme.

(b) Given the infinitesimal translation operator $\hat{T}(dx) = 1 - i\,\hat{p}\,dx/\hbar$ show that momentum operator in position basis is $\hat{p} = -i\hbar\frac{\partial}{\partial x}$. Show that

$$\langle x|p\rangle = N\exp\left(\frac{i\,p\,x}{\hbar}\right)$$

Find N and show that $\langle x|\psi\rangle$ and $\langle p|\psi\rangle$ are fourier transform pairs. (9+9.75)

3. (a) An infinitesimal translation $\hat{T}(dx)$ takes position eigenket $|x\rangle$ to $|x+dx\rangle$. For simplicity take translation in one dimension. If the state vector is normalized to unity show that this translation forms a group. Show that $\hat{T}(dx)=1-i\hat{k}dx$ is right choice and forms a unitary group.

(b) Find the commutation relation and uncertainty between position and momentum operators. The generalized uncertainty relation is given as

$$\left\langle (\Delta\hat{A})^2 \right\rangle \left\langle (\Delta\hat{B})^2 \right\rangle \geq \frac{1}{4} \left| \left\langle [\hat{A}, \hat{B}] \right\rangle \right|^2$$

where $\Delta\hat{A} = \hat{A} - \left\langle \hat{A} \right\rangle$, and each term has their conventional meaning. (10+8.75)

4. (a) Write in brief the difference between Schrodinger and Heisenberg Picture. How are the states, operators, eigenvalues and expectation values of one picture related with the respective quantities in other picture?

(b) Consider a system of two identical particles. A general state of this system can be written as $|u\rangle = \alpha|u_i(1), u_j(2)\rangle + \beta|u_j(1), u_i(2)\rangle$. Construct the physical (orthonormal) states of the system as advised by symmetrization postulate. Generalize this postulate to N particle system and show that a completely antisymmetric state can be written in the form of a determinant, called Slatter determinant. (8+10.75)

5. Consider the case where $j = 1$.

(a) Find the matrices representing the operators $\hat{J}^2, \hat{J}_z, \hat{J}_x, \hat{J}_y$ and $\hat{J}_\pm$.

(b) Find the joint eigenstates of $\hat{J}^2$ and $\hat{J}_z$ and verify that they form an orthonormal and complete basis.

(c) Use the matrices of $\hat{J}_x$, $\hat{J}_y$ and $\hat{J}_z$ to calculate $[\hat{J}_x, \hat{J}_y]$, $[\hat{J}_y, \hat{J}_z]$, and $[\hat{J}_z, \hat{J}_x]$.

(d) Verify that $\hat{J}_z^3 = \hbar^2 \hat{J}_z$ and $\hat{J}_\pm^3 = 0$.

(4+4+6+4.75)

6. (a) Consider a system with Hamiltonian as follows

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

where $\delta(x)$ is Dirac Delta function. Use a gaussian trial function $\psi(x) = Ae^{-bx^2}$ where b and A are constant, and A is normalizing factor. Calculate A and b such that the $\langle \hat{H} \rangle$ is minimum.

(b) Find an upper bound on the ground state energy of the one dimensional infinite square well using the following trial wave function :

$$\psi(x) = Ax, \quad \text{if } 0 \le x \le a/2,$$

$$= A\,(a - x), \quad \text{if } a/1 \le x \le a,$$

$$= 0 \quad \text{Otherwise}$$

(9+9.75)